THE
ECONOMIC
JOURNAL

*The Economic Journal*, Doi: 10.1111/ecoj.12203 © 2014 Royal Economic Society. Published by John Wiley & Sons, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

# LOOPHOLES*

*Kim-Sau Chung and Lance Fortnow*

We formalise the argument of those American founding fathers who opposed the inclusion of the Bill of Rights into the Constitution. For some parameter values, the legislator, who is not sure whether or not there are any rights that he is unaware of, optimally chooses not to enumerate even those rights that he is aware of. We also show that, even if the legislator can add the sentence 'this Bill should not be interpreted as suggesting that any unlisted rights can be impaired by the government' to the Bill, the equilibrium outcome will stay the same.

This study revisits an old debate among America's founding fathers, namely whether or not the Bill of Rights should be included in the Constitution. Some of the founding fathers, e.g. James Iredell, subsequent Supreme Court Justice, strongly opposed the inclusion. Iredell told his fellow constitution ratifiers in North Carolina that it would be 'not only useless, but dangerous, to enumerate a number of rights which are not intended to be given up; because it would be implying, in the strongest manner, that every right not included in the exception might be impaired by the government without usurpation' (Elliot, *Debates*, 167 (James Iredell, North Carolina ratifying convention, Tuesday, July 29, 1788)). The goal of this study is to examine the logic behind Iredell's argument, and explore its relations with incomplete contracts.

As is now well known, Iredell's argument did not prevail, and the Bill of Rights was eventually included in the American Constitution as a series of amendments. One may argue that Iredell's argument did not prevail because it contained a serious logical hole: if Iredell was worried that any omitted rights would be made more vulnerable by a detailed but inevitably incomplete Bill of Rights, then a better way to address his concern would be to write explicitly in the Bill that any omitted rights should be deemed as equally sacred as those rights in the Bill. More generally, a more direct way to address his concern would be to express it explicitly in the Bill, by, for example adding the following sentence: 'This list of rights is not meant to be exhaustive, and is limited by our own awareness, and hence this Bill should not be interpreted as suggesting that any unlisted rights can be impaired by the government'. Why would Iredell's recommended action (i.e. not to write the Bill at all) ever be optimal?

Understanding Iredell's logic can teach us something that goes beyond this isolated historic event. In particular, Iredell's argument resembled many modern-day arguments why it is sometimes optimal to write incomplete contracts. For example, merger agreements usually contain a material-adverse-change (MAC) clause that allows either party in a merger to opt out before completing the deal. The language of the

* Corresponding author: Kim-Sau Chung, Department of Economics, Chinese University of Hong Kong, Shatin, Hong Kong. Email: kschung@cuhk.edu.hk.

We thank Oliver Board, Jiahua Che, Yuk-shing Cheng, Peter Eso, Chia-hui Lu and the audiences at various seminars/conferences for helpful comments. The work described in this article was partially supported by grants from the Research Grants Council of the Hong Kong Special Administrative Region, China (CUHK-490513) and from CUHK (Direct Grant 4052015).

clause is typically vague, leaving it to the courts to decide what it means by a 'material' adverse change (that damages one party's business enough to justify the other party's pulling out). Why is it not a good idea to make the MAC clause less vague? *The Economist* magazine explains: 'If a clause is too specific, factors that are not cited explicitly may be assumed by the courts to be excluded'.[1] Note the resemblance between this argument and that of Iredell.

According to an urban legend among economists, once upon a time there was a very intelligent economist called Sanford, who signed a contract with a subcontractor to build a new house. Being a non-trusting home owner, Sanford wrote an extremely detailed contract, painstakingly enumerating many requirements for his new house. When the house was finished, he found the house to be defective – perhaps the roof was leaking – something that had somehow slipped his mind when he wrote his otherwise very detailed contract. He refused to pay the subcontractor because of this defect but lost the case in court. The judge explained that a leaky roof would typically be considered as unacceptable. However, since Sanford's contract was so detailed and yet did not require a good roof, the judge reasoned, he must have thought about the possibility of a leaky roof and decided that it was acceptable. Sanford's story is one of economists' favourite classroom stories to explain the merit of incomplete contracts.

In the light of the apparent logical hole in Iredell's argument, we can similarly challenge these arguments for incomplete contracts. Would not companies do even better if they replace those vague MAC clauses with detailed lists of opt-out excuses, capped with an extra clause saying that those lists are not meant to be exhaustive, and are limited by the contracting parties' awareness, and hence should not be interpreted as suggesting that any unlisted excuses are invalid? Would Sanford have saved himself from the unnecessary agony if he had added to his already very detailed contract an extra sentence saying that his list of requirements was not meant to be exhaustive, and was limited by his own awareness, and hence should not be interpreted as suggesting that any unlisted requirements were not important? Why and why not?

This article formally examines Iredell's argument by studying a dynamic game with two players: a legislator who is to write the Bill of Rights and a judge who is to interpret it 200 years later. The legislator is aware of certain rights, which he can include in the Bill but is also unaware of some other rights. He is aware of his own unawareness but is uncertain about the number of rights he is unaware of.

We show that, for some range of parameter values, there is a unique equilibrium where the legislator optimally chooses not to write the Bill of Rights at all – that is, not even to enumerate those rights that he is aware of. The reason is that, in equilibrium, how the judge treats those rights not in the Bill depends on how elaborate the Bill is. The more elaborate the Bill is, the less likely that the judge would protect those unlisted rights.

More importantly, we also prove that, even if the legislator adds the sentence 'any other rights not listed in this Bill are equally sacred and the government should not infringe them either' to the Bill, the equilibrium outcome will stay the same (Theorem 2).

[1] *The Economist,* 8 December 2001, p. 58.

These two results, combined together, suggest that the logical hole in Iredell's argument was purely illusionary. Other founding fathers might disagree with Iredell on the values of certain parameters but it would be wrong to dismiss his argument as illogical.

Anyone who tries to model and predict a judge's behaviour must make heroic assumptions on the judge's preferences and constraints. In our model, a crucial assumption is that judges are not constrained to adhere strictly to the legal interpretive doctrine of *expressio unius est exclusio alterius* (hereafter the *expressio doctrine*). This is probably the most controversial assumption in this article and hence we shall devote a whole Section (Section 2) to defending it. Indeed, we would even take the stand that debunking the myth of the *expressio doctrine* and formally modelling how judges deviate from it, holds the key of understanding how the common law system works.

Translated into English, the *expressio doctrine* says 'expression of the one is exclusion of the other'. For example, if a law says 'children below 16 are not allowed to drive', then we do not need to ask whether a 17-year-old girl is allowed to drive or not. She is. However, as we argue in Section 2, the *expressio doctrine* is merely a myth and judges do not adhere strictly to it. Section 2 also informally suggests why and how judges deviate from it, which we subsequently incorporate into our formal model.

Roughly speaking, judges deviate from the *expressio doctrine* by first performing an 'awareness check': instead of immediately jumping to the conclusion that a 17-year-old girl is allowed to drive, they first ask whether the legislators had likely thought about the case of a 17-year-old girl. If not, then the judges's ruling is no longer bound by the *expressio doctrine*. In Section 2, we use two famous Supreme Court cases to illustrate this systematic deviation from the *expressio doctrine*.

When judges decide that they are not bound by the *expressio doctrine*, they exercise their professional judgment to rule on the case. Exercising one's professional judgment is costly and judges are willing to do so only when the expected improvements in the quality of their rulings are big enough. Since expected improvements depend on whether the legislators have high or low awareness types (we shall be more precise about what we mean by 'awareness types' shortly), judges perform an 'awareness check' before deciding whether to exercise their professional judgment. Anticipating that judges will behave in this way, when the legislators want to incentivise the judges to exercise their professional judgment, they write a more incomplete law and credibly signal to the judges that they have low awareness types. This is the gist of Iredell's argument.

It also explains why Iredell might not be able to address his concern by merely adding an other-rights-are-also-sacred clause to the Bill of Rights, or why companies may not gain extra flexibility by merely adding an other-opt-out-excuses-are-also-valid clause to a merger agreement, or why Sanford might not be able to better protect himself by merely adding an other-requirements-are-also-important clause to his already very detailed contract. The purpose of these clauses, if they are ever effective, is to affect the results of the judges' 'awareness check', and increase their incentives to exercise their professional judgment. But these clauses are not credible signalling devices and do not affect the results of 'awareness check'.

The next Section reviews the related literature. Section 2 defends our assumption that judges do not adhere strictly to the *expressio doctrine*. Sections 3-5 present the model and our main result. Sections 6 discusses the relations with the literature of incomplete contracts.

## 1. Related Literature

Since laws can be viewed as (social) contracts among citizens, our article is naturally related to the literature of strategic incompleteness in contracts (Spier, 1992; Bernheim and Whinston, 1998; Tirole, 2009; Bolton and Faure-Grimaud, 2010; Ederer *et al.*, 2011). To us, the more important difference between our article and these previous studies is that we have different definitions of contract incompleteness in mind. If the definitions of contract incompleteness are different, the models are *a fortiori* different. It is, however, easier to postpone the delineation of these different definitions of contract incompleteness to Section 6 after we present our main results.

Here, we merely point out one apparent difference in the models. As is typical in contract theory, an unspoken assumption in this literature is that the judge (who often does not even appear in the model) interprets any contract to the letter. In the special case of social contracts, this unspoken assumption would correspond to the assumption that judges adhere to the *expressio doctrine* – the very assumption that we vehemently refute in this article. Indeed, we see as one of our contributions to provide an alternative theory of how judges actually behave. This (endogenous) behaviour on the part of judges is at the heart of our theory of strategic incompleteness.

The particular setting of social contracts also renders some previous theories of strategic incompleteness inapplicable without appropriate modification. For example, in Bernheim and Whinston (1998) and Ederer *et al.* (2011), contract incompleteness allows the principal to exercise discretion and respond to the agent's actions. But the American founding fathers would be long dead before future citizens can take any action. One way to modify these theories is to bring in a future judge to serve as a surrogate principal. However, such a modification will work only if we at the same time eschew the assumption that judges are constrained to adhere to the *expressio doctrine* – an assumption that we argue is more a myth than realistic. In this sense, our article and previous theories are complementary.

Jehiel and Newman (2012) provide an interesting theory of why we often observe 'loopholey' contracts. Contracts that have no loophole are costly in good states where cheating is infeasible. But if 'loopholey' contracts do not appear in bad states where cheating is feasible, the society can never learn whether good states have arrived or not. Therefore, 'loopholey' contracts that get cheated on serve a positive role in society, and will also be offered in equilibrium if social learning involves a little bit of imperfection. Jehiel and Newman's (2012) 'loopholey' contracts are contracts with insufficient details to deter cheating. In our article, 'loopholey' contracts are contracts with excessive details, such as a merger contract that replaces the standard MAC clause with a detailed list of material adverse changes.

Our article is also related to the legal literature on rules versus standards. In Ehrlich and Posner (1974), rules are understood as precise boundaries between good and bad behaviour, whereas standards correspond to noisy boundaries. The noisiness inherent in a standard is costly to risk-averse citizens but the precise boundary described in a rule may also be a poorer approximation to society's ideal boundary. So a choice between rules versus standards depends on this tradeoff.

Kaplow's (1992) analysis of rules versus standards is closer to our article. In Kaplow (1992), standards are similar to what we will call 'barebone laws' in our model, whereas rules are more elaborate descriptions of good and bad behaviour. Writing rules incurs more *ex ante* transaction costs but saves on *ex post* litigation costs. The choice between rules and standards balances these two kinds of costs.[2] Kaplow's (1992) theory is not suitable to analyse Iredell's dilemma. When Iredell opposed the inclusion of the Bill of Rights in the Constitution, the Bill had already been written and put on the table. Any *ex ante* transaction costs, such as the costs of deliberation and enumeration, or the costs of carefully crafting the delicate wording, were already paid. By opposing the inclusion, Iredell saved none of these costs. Nor did Iredell use any cost-saving argument similar to Kaplow's (1992) to persuade his fellow founding fathers. If anything, the very act of opposing the inclusion was costly to Iredell personally, and in that sense Iredell does not even pass the first sanity test in Kaplow's (1992) world.

## 2. Debunking the Myth of the *Expressio Doctrine*

As mentioned in the Introduction, a crucial assumption in our model is that judges are not constrained to adhere strictly to the legal interpretive doctrine of *expressio doctrine*. Since this assumption may be controversial, we shall devote this whole Section to defending it. In particular, we shall use two famous Supreme Court cases to demonstrate:

(*i*) not only that Justices have no problem deviating from the doctrine, moreover;
(*ii*) they deviate from it in a systematic way.

That systematic way is to perform an 'awareness check' first, and to adhere to the doctrine only if it seems likely that the legislators have thought about cases similar to the one the Justices are considering. In Section 3, this idea of 'awareness check' will be incorporated into our formal model.

### 2.1. *Maryland versus Craig*, 497 *US* 836 (1990)

This is a case involving a prosecution for sexual abuse of a young child. When the case was tried in the trial court, the judge decided that the child would be too frightened to testify in the presence of the presumed abuser and hence allowed her to testify in a separate room, with only the prosecutor and defence counsel present, while the defendant, the judge and the jury watched over closed-circuit television. The defendant challenged the constitutionality of this procedure and the case went all the way to the Supreme Court.

According to the Confrontation Clause of the Sixth Amendment of the American Constitution, '[i]n all criminal prosecutions the accused shall enjoy the right ... to be confronted with the witnesses against him'. The defendant claimed that the procedure used in the trial court violated this clause. The Supreme Court split 5 to 4 on this issue,

[2] See Scott and Triantis (2006) for a similar theory.

showing that this was by no means a clear-cut case. It is illuminating to examine in detail where the disagreement laid.

The disagreement was not over what 'confrontation' meant. Both sides agreed that it means 'face-to-face', in particular not 'watching from another room'. There was also no disagreement on why there was such a provision in the Constitution in the first place. It was agreed that the major purpose of this provision was precisely to frighten witnesses to discourage them from lying.

More relevant to our claim that strict adherence to the *expressio doctrine* is a myth, there was also agreement that 'all' did not literally mean 'all'! The majority opinion summarised the core question of this case as follows: '[The question is] whether any exceptions exist to the irreducible literal meaning of the Clause ... [i]t is all but universally assumed that there are circumstances that excuse compliance with the right of confrontation'. Since 'all' did not literally mean 'all', the Justices were now left to debate on whether a child-sexual-abuse case belonged to the set 'all' or to its complement – and this was where the disagreement laid.

How did the Justices decide whether a child-sexual-abuse case belonged to the set 'all' or to its complement? The answer is: by an 'awareness check'. For example, Justice Scalia, who wrote the minority opinion, emphasised the following observations when he defended his opinion later: 'Sexual abuse existed [in 1791, the time of the Sixth Amemdment], as it does now; little children were more easily upset than adults, then and now; a means of placing the defendant out of sight of the witness existed then as now (a screen could easily have been erected that would enable the defendant to see the witness, but not the witness the defendant)' (Scalia, 1998). After highlighting these observations, Justice Scalia asserted that a child-sexual-abuse belonged to the set 'all', and hence the procedure used in the trial court was unconstitutional. Why were these observations important? Although Justice Scalia did not explain, it is only natural to guess that he considered them important because they increased the likelihood that the authors of the Sixth Amendment were aware of the cases of sexual abuse of young child. The higher is that likelihood, the more plausible is the hypothesis that a child-sexual-abuse case belongs to the set 'all'.[3]

### 2.2. *Church of Holy Trinity versus United States*, 143 *US* 457 (1892)

The Church of Holy Trinity, located in New York City, contracted with an Englishman to come over to be its pastor. The government claimed that this agreement violated a federal statute that made it unlawful for any person to 'in any way assist or encourage the importation or migration of any alien ... into the United States ... under contract or agreement ... made previous to the importation or migration of such alien ... to perform labor or service of any kind in the United States'. The fifth Section of the statute makes specific exceptions, among them professional actors, artists, lecturers, singers and domestic servants; but the exceptions notably do not include pastors.

[3] Given this logic, Justice Scalia found the majority opinion unfathomable. Rhetorics aside, he should not have. After all, his observations only increase the likelihood that a child-sexual-abuse case belongs to the set 'all'; reasonable people can still disagree on whether that likelihood is big enough after the increase.

The case went all the way to the Supreme Court and the Court ruled in favour of Holy Trinity. Had the Justices adhered strictly to the *expressio doctrine*, this would have been a clear-cut case: Holy Trinity violated the law. However, in a now famous quote, the Court in effect announced that it did not plan to adhere strictly to the *expressio doctrine*: 'It is a familiar rule, that a thing may be within the letter of the statute and yet not within the statute'. The Court then, famously, devoted seven pages of its opinion to a lengthy discussion of how America is a religious nation. From that discussion, it concluded that '[i]t is a case where there was presented a definite evil, in view of which the legislature used general terms with the purpose of reaching all phases of that evil, and thereafter, unexpectedly, it is developed that the general language thus employed is broad enough to reach cases and acts which the whole history and life of the country affirm could not have been intentionally legislated against'.

How should we make sense of this ruling? There is a malign interpretation: the Court had simply abused its power. According to this interpretation, the Justices were religious people and they argued in seven pages that every other citizen was as religious as they were; hence they insisted that granting churches preferential treatment was more important than following the law.

The malign interpretation may well be the correct one but there is also a benign interpretation. According to this interpretation, the Court was entertaining two competing hypotheses: the one that pastors truly do not belong to the exception list; and an alternative one that legislators were simply absent-minded and omitted examples such as pastors – had someone brought these examples to their attention they would have included them in the list. Since 'America is a religious country', the Court regarded the first hypothesis as less likely. And given this result of their 'awareness check', the Justices decided that they were no longer bound by the *expressio doctrine*.

## 3. The Model

There are two players in the game: the legislator ($L$, who we assume to be male), and the judge ($J$, who we assume to be female). For concreteness, we can think of the legislator is to write the Bill of Rights, and the judge is to interpret it 200 years later. In period 1, $L$ decides how to write a law. He dies at the end of period 1. In period 2, nature randomly chooses an action, which we can think of as the government infringing a particular right. In period 3, that randomly chosen action is in front of $J$, and she has to rule whether or not it is illegal.

### 3.1. *Actions*: *the Good, the Bad and the Fair*

Being the representative of the people, $L$'s own personal preferences define what are good and what are bad. One reason why communication of these preferences to $J$ is imperfect is that $L$ may not be aware of every possible action at the time of communication (i.e. at the time when he writes the law).

For every action, if $L$ were aware of it, he would have regarded it as either good or bad. Among all possible actions, $L$ would have regarded $n$ of them good and $m$ of them bad. So there are totally $n + m$ possible actions.

Nature picks one of these $n + m$ actions using the following conditional probability distribution. With probability 1/2, nature picks a good action; and with probability 1/2 a bad action. Among the good actions, each has equal probability of being picked; similarly for bad actions. So every good action has probability $1/2n$ being picked and every bad action $1/2m$.

The sets of good and bad actions are asymmetric: $n$ is a fixed number which we assume is very large (or, equivalently, $1/n$ is vanishingly small); while $m$ is random and is either 1 or 2 with equal probability. This asymmetry is necessary to explain the phenomenon that real-life laws are often lists of bad behaviour instead of lists of good behaviour.

$J$ does not know $L$'s personal preferences. However, we assume the existence of an exogenous technology, using which $J$ can figure out whether a particular action is fair or unfair. We call this technology the `fairness test`. We will see very soon why we chose this terminology, but it is meant to model nothing more than a judge exercising her professional judgment to fill in any gap she conceives in the law. A good action will be found fair by the test with probability $p$, where $1/2 < p < 1$. Symmetrically, a bad action will be found unfair with probability $p$ as well.

### 3.2. *Legislator: High and Low Types*

A blank piece of paper is not the minimal form of law, because it does not give the court jurisdiction over actions, and in that sense it is not even a law. A barebone law contains at least one sentence: 'All actions that are unfair are hereby declared illegal'. This gives the court jurisdiction over actions. Given such a barebone law and given any action randomly chosen by nature in period 2, $J$ is not bound by this law to employ the costly `fairness test`. She can still short-circuit the costly `fairness test` and rule the action `legal` or `illegal` right away. So the only difference between the barebones law and a blank piece of paper is that the former gives the court jurisdiction. Any law that $L$ may choose to write in period 1 must be at least barebones.

The barebone law best known to economists is perhaps the US FTC Act, which states that: 'The [Federal Trade Commission] is empowered and directed to prevent persons, partnerships or corporations ... from using *unfair* methods of competition in or affecting commerce'. The word 'unfair' is famously left undefined in the FTC Act.

On top of a barebones law, $L$ can choose to add exceptions; for example 'Congress shall make no law abridging the freedom of speech' or 'Price fixing is hereby declared *per se* illegal'. Formally, an exception is a pair of the form (`action`, `good`) or (`action`, `bad`). The former (latter) reads as: 'Action `action` is hereby declared *per se* legal (illegal)'.

Adding each exception incurs a cost of $c$, which we assume is small, but larger than $1/n$.

In order to add the exception (`action, good` or `bad`), $L$ must be at least aware of action `action`. If $L$ were aware of all $m + n$ actions, he could have listed all of them as exceptions (with appropriate `good` or `bad` labels), and then there would be no guesswork left for $J$ to do in period 3. But no legislator can have that extreme level of awareness.

To fix the idea, imagine that $L$ is writing the law under some time pressure. After thinking really hard within a certain time limit, $L$ can still only think of a small portion of all possible actions. He is painfully aware of his limitation and, in particular, he is fully aware of the fact that there are still a lot more actions out there that he has not yet thought of. But since time is up, he has to start writing the law now. His choice of exception list is hence limited by the number of actions that he has thought of. We assume that $L$ can be of either high or low type. A high-type $L$ is able to think of two actions when time is up and a low-type $L$ is able to think of only one. $L$ has a high type with probability $v \in (0,1)$. The exact magnitude of $v$ is not important in our analysis.

For a low-type $L$, the single action that he can think of is a random selection from the set of all actions and the distribution is the same as the distribution with which nature later on picks an action in period 2. That is, conditional on $m$ and $n$, each good action has a probability of $1/2n$ being thought of and each bad action $1/2m$. The assumption that these two distributions are the same is not important. It is made only to ease notation.[4]

For a high-type $L$, the two actions that he can think of are drawn from the same distribution without replacement.

We can think of a typical information set of a high-type $L$ takes the form of {(`xxx`, good), (`yyy`, bad)}; that is, he has thought of one good action (action `xxx`) and one bad action (action `yyy`). Whether to add exceptions is $L$'s discretion. In particular, he is not required to add either the exception (`xxx`,`good`) or the exception (`yyy`, `bad`). However, he cannot add an exception such as (`zzz`, `bad`), because he is not aware of action `zzz`. Similarly for a low-type $L$.

In this model, we also assume that $L$ cannot lie about his preferences. For example, he cannot add the exception (`xxx`, `bad`) or (`yyy`, `good`) if his information set is {(`xxx`, `good`), (`yyy`, `bad`)}. In an earlier version of this article, we do not forbid $L$ from lying, and the results are the same (in equilibrium, he will never lie), while the analysis is much messier.

Strictly speaking, {(`xxx`, `good`), (`yyy`, `bad`)} and {(`xxx`, `good`), (`zzz`, `bad`)} are two different information sets of a high-type $L$ and they result in two different feasible choice sets for $L$. However, specific names of actions are not important in our analysis. We shall hence simplify notations by representing both information sets as BG, meaning that $L$ has thought of one good and one bad action. We abuse terminology and continue to call BG an information set of a high-type $L$. Therefore, a high-type $L$ has three possible information sets: BB, BG and GG. Similarly, a low-type $L$ has two possible information sets: B and G. We shall use $\Phi$ = {BB, BG, GG, B, G} to denote the set of all possible information sets, with $\phi$ a typical information set.

Similarly, for the purpose of our analysis, we do not need to distinguish a law with exceptions {(`xxx`, `bad`),(`yyy`, `bad`)} and another with exceptions {(`zzz`, `bad`),(`yyy`, `bad`)}. We shall simplify notations by representing both laws as a `bb` law, meaning that it contains two exceptions, naming two different actions as bad. We define other forms of laws (`bg`, `gg`, `b` and `g`) similarly. And we use $\varnothing$ to denote a barebone law. The feasible choice set of $L$ given each possible information set is hence as shown in Table 1.

[4] One possible justification of this assumption is that it may be more likely that a legislator is aware of an action if that action is taken more frequently.

Table 1
*Feasible Choice Sets of L*

| Information sets | Feasible choice sets of $L$ |
|---|---|
| BB | {bb, b, ∅} |
| BG | {bg, b, g, ∅} |
| GG | {gg, g, ∅} |
| B | {b, ∅} |
| G | {g, ∅} |

$L$'s strategy $\sigma_L$ is a mapping that assigns to each possible information set a choice in his feasible choice set.

We shall use LAWS = {bb, bg, gg, b, g, ∅} to denote the set of all possible laws, with law a typical law. In Section 5, we shall extend the set LAWS to accommodate extra sentences such as 'this list of actions is not meant to be exhaustive, and is limited by the legislator's awareness'.

### 3.3. *Judge: To Deliberate or not to Deliberate*

After nature randomly chooses an action in period 2, $J$ is to rule on its legality in period 3. We assume that, if nature chooses action action, and if there is an exception (action, good) in the law, then $J$ is required to rule action action as legal. Similarly, if there is an exception (action, bad) in the law, then $J$ is required to rule action action as illegal. So, there is a non-trivial decision for $J$ to make only when the chosen action is not mentioned in (any of the exceptions in) the law.

When the chosen action is not mentioned in the law, $J$ has three options. She can rule it as legal right away, or she can rule it as illegal right away. Neither of these options incurs any effort cost. Her third option is to employ the fairness test, and then rule the action as legal if it is found fair, and as illegal if it is found unfair.[5] Employing the fairness test incurs effort cost $e$. Implicitly, we are assuming that whether a judge makes her ruling with or without serious deliberation is unobservable to the rest of the society, and hence a judge cannot be mandated to engage in serious deliberation (which is costly to her) before making her ruling.

$J$'s strategy is hence a mapping $\sigma_J$ from the set of all possible laws LAWS to the three options {legal, illegal, fairness test}.

### 3.4. *Preferences*

$J$'s objective has two parts. The first part is to serve $L$ (who represents the people): if she knows $L$'s personal preferences and hence which actions are good and which are bad, she would like to rule a good action as legal and a bad action illegal. The second part is to minimise the effort cost associated with deliberaztion.

[5] We can also allow $J$ to have discretion on how to rule according to the result of the fairness test, but the result will be the same.

To formalise this, we define the 'loss from judicial errors' as 1 if $J$ rules a good action as `illegal` or a bad action as `legal`; and the loss is 0 otherwise. $J$'s utility function is hence

$$U_J = -\mathbb{E}(\text{loss from judicial errors}) - \mathbb{I}_f e,$$

where $\mathbb{I}_f$ is an indicator function equal to 1 if $J$ employs the `fairness test` and equal to 0 otherwise.

Let $\alpha(\texttt{law})$ be $J$'s belief that the action chosen by nature is a good action, conditional on that the chosen action is not mentioned in the law `law`.[6] If $J$ rules the chosen action as `legal` (respectively, `illegal`) right away, the expected loss from judicial errors will be $1-\alpha(\texttt{law})$ (respectively, $\alpha(\texttt{law})$). If $J$ employs the `fairness test` and then rule accordingly, the expected loss from judicial errors will be $\alpha(\texttt{law})(1-p)+[1-\alpha(\texttt{law})](1-p)=(1-p)$. Therefore, $J$'s utility function can be rewritten as

$$U_J = -\mathbb{I}_{\texttt{legal}}[1-\alpha(\texttt{law})] - \mathbb{I}_{\texttt{illegal}}\alpha(\texttt{law}) - \mathbb{I}_f[(1-p)+e],$$

where $\mathbb{I}_{\texttt{legal}}$ and $\mathbb{I}_{\texttt{illegal}}$ are the indicator function for ruling the chosen action as `legal` and `illegal`, respectively, right away.

We shall use $l$ to denote the sum $(1-p)+e$, where $l$ stands for the total litigation cost associated with the `fairness test`. Apparently, the `fairness test` will be redundant in this model if $l > 1/2$, because the judge can guarantee an expected loss from judicial error of no more than 1/2 by making a ruling right away. Therefore, we focus on the case where $l < 1/2$.

$L$'s objective also has two parts. The first part is to minimise the period-1 law-writing cost, and second part is to minimise the period-3 expected loss from judicial errors. His utility function is hence

$$U_L = -c \times (\text{number of exceptions}) - \mathbb{E}(\text{loss from judicial errors}).$$

In this model, the legislator does not internalise the judge's effort cost associated with deliberation. This misalignment of preferences is important to our result.[7] An equivalent assumption is that the judge does not fully internalise the social benefit of making a correct ruling. One may wonder why the judge would seem selfish in this sense, while at the same time selfless in the sense that she bases her ruling on her inference of the legislator's opinions on what is good and what is bad behaviour. In a more realistic model with more than one judge and hence potential free-riding problem among judges, this particular form of agency problem may arise more naturally.

### 3.5. *Solution Concept*

The game described above is a finite dynamic game with incomplete information. We can hence solve for the sequential equilibria satisfying the intuitive criterion (Kreps and Wilson, 1982; Cho and Kreps, 1987). The set of sequential equilibria not

[6] This belief of course also depends on $J$'s knowledge of $L$'s strategy.

[7] If the legislator internalises the judge's effort cost associated with deliberation, the two players' preferences will be perfectly aligned in the continuation game starting from period 2. Therefore, when writing cost goes to 0, the legislator will write into the law every bad action he is aware of and trust that the judge will in period 3 take an action that is the best for both of them (whose preferences are the same). In other words, we will not have any equilibrium where the legislator deliberately refrains from writing into the law a bad action that he is aware of.

surprisingly depends on $n$ (the number of good actions) and $c$ (the cost of adding each exception in the law). We, however, focus on the case where $1/n$ is much smaller than $c$, while $c$ is also very small. Intuitively, when $1/n$ is much smaller than $c$, $L$ will not write any good action into the law and hence any law arising in sequential equilibrium will resemble those we observe in the real life and is an enumeration of bad behaviour only. On the other hand, if $c$ is also very small, then we can convincingly argue that the reason why a legislator may not write all bad actions he is aware of into the law must be something other than writing cost.

To focus on this limit case, we define the following solution concept. Recall that any sequential equilibrium is an assessment, which in this game takes the form of $(\sigma_L, \sigma_J, \beta_J)$, where $\beta_J$ is $J$'s probabilistic belief of $L$'s information set given the law written by $L$ (i.e. $\beta_J$ is a mapping from `LAWS` to probability distributions over $\Phi$).

Definition 1. *An equilibrium is a pure strategy profile* $(\sigma_L^*, \sigma_J^*)$ *such that there exist* $\bar{c} > 0$ *and* $r > 0$ *such that, for any* $c \in (0, \bar{c})$ *and any* $n$ *such that* $1/cn \in (0,r)$, *there exists a sequential equilibrium* $(\sigma_L, \sigma_J, \beta_J)$ *satisfying the intuitive criterion such that* $(\sigma_L, \sigma_J) = (\sigma_L^*, \sigma_J^*)$.

Note that according to the above definition any equilibrium must be a profile of pure strategies.

## 4. Equilibria

In Table 2, we list the joint probability of each $(m,\phi)$, as well as the conditional probability of $m = 2$ conditional on each $\phi$.

Using $\Pr(m = 2|\phi)$, we can calculate the conditional probability $q(\texttt{law}, \phi)$ that the action chosen by nature is good, conditional on that the action is not mentioned in the law `law`, and that $L$'s information set is $\phi$. For example, conditional on that the chosen action is not mentioned in the law `bg`, and that $L$'s information set is BG, the conditional probability $q(\texttt{bg}, \text{BG})$ that the action chosen by nature is good is:

Table 2
*Joint Probability and Conditional Probability*

| $\phi$ | $\Pr(m = 1,\phi)$ | $\Pr(m = 2,\phi)$ | $\Pr(m = 2\|\phi)$ |
|---|---|---|---|
| BB | 02 | $\frac{v}{2}\left(\frac{1}{2} \times \frac{1}{3}\right)$ | 1 |
| BG | $\frac{v}{2}\left(\frac{1}{2} + \frac{1}{2} \times \frac{1/2}{1-1/2n}\right)$ | $\frac{v}{2}\left(\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1/2}{1-1/2n}\right)$ | $\frac{7n-2}{16n-5}$ |
| GG | $\frac{v}{2}\left[\frac{1}{2} \times \frac{(n-1)/2n}{1-1/2n}\right]$ | $\frac{v}{2}\left[\frac{1}{2} \times \frac{(n-1)/2n}{1-1/2n}\right]$ | $\frac{1}{2}$ |
| B | $\frac{1-v}{2} \times \frac{1}{2}$ | $\frac{1-v}{2} \times \frac{1}{2}$ | $\frac{1}{2}$ |
| G | $\frac{1-v}{2} \times \frac{1}{2}$ | $\frac{1-v}{2} \times \frac{1}{2}$ | $\frac{1}{2}$ |

$$q(\texttt{bg}, \text{BG}) = \Pr(m = 1|\text{BG}) \times 1 + \Pr(m = 2|\text{BG}) \times \frac{(n-1)/2n}{1 - \frac{1}{4} - \frac{1}{2n}}$$

$$= \frac{41n^2 - 45n + 10}{(16n - 5)(3n - 2)}.$$

Other conditional probabilities can be calculated similarly, and are listed in Table 3 (blank cells correspond to impossible combinations of ($\texttt{law}$,$\phi$)).

Recall that $\alpha(\texttt{law})$ is $J$'s belief that the action chosen by nature is good, conditional on that it is not mentioned in the law $\texttt{law}$; whereas $\beta(\texttt{law})$ is $J$'s probabilistic belief about $L$'s information set, given that $L$ has written the law $\texttt{law}$. These two beliefs are related by the following equation:

$$\alpha(\texttt{law}) = \sum_{\phi \in \Phi} \beta(\texttt{law})(\phi) \times q(\texttt{law}, \phi).$$

In particular, $\alpha(\texttt{law})$ is a convex combination of the $q(\texttt{law}, \phi)$s for different $\phi$s. This immediately implies that $\alpha(\texttt{bb}) = 1$, $\alpha(\texttt{gg}) = (n-2)/(2n-2)$, $\alpha(\texttt{g}) = (n-1)/(2n-1)$ and $\alpha(\varnothing) = 1/2$ in any sequential equilibrium.

LEMMA 1. *In any equilibrium* $(\sigma_L^*, \sigma_J^*)$, *we must have* $\sigma_J^*(\texttt{bb}) = \texttt{legal}, \sigma_J^*(\texttt{gg}) = \sigma_J^*(\texttt{g}) = \sigma_J^*(\varnothing) = \texttt{fairness test}$, *and* $\sigma_J^*(\texttt{law}) \neq \texttt{illegal}$ *for any other law* $\texttt{law}$.

All proofs can be found in the Appendix.

LEMMA 2. *In any equilibrium* $(\sigma_L^*, \sigma_J^*)$, *we must have* $\sigma_L^*(\text{GG}) = \sigma_L^*(\text{G}) = \varnothing$ *and* $\sigma_L^*(\text{BB}) = \texttt{bb}$.

Lemma 2 is very intuitive. Given that there are a lot of good actions and each will appear in the court with vanishingly small probability, being able to think of one or two good actions is pretty useless as far as law making is concerned. So a legislator with information set GG or G may as well write a barebones law, and let the judge exercise her professional judgment on the bench. On the other extreme, a legislator who has thought

Table 3
*$q(\texttt{law},\phi)$*

| $q(\texttt{law},\phi)$ | bb | bg | gg | b | g | $\varnothing$ |
|---|---|---|---|---|---|---|
| BB | 1 | | | 2/3 | | 1/2 |
| BG | | $\frac{41n^2-45n+10}{(16n-5)(3n-2)}$ | | $\frac{41n-13}{3(16n-5)}$ | $\frac{n-1}{2n-1}$ | 1/2 |
| GG | | | $\frac{n-2}{2n-2}$ | | $\frac{n-1}{2n-1}$ | 1/2 |
| B | | | | 5/6 | | 1/2 |
| G | | | | | $\frac{n-1}{2n-1}$ | 1/2 |

of two bad action has already thought of all bad actions, so there is no reason not to write all of them into the law, and let the judge rule every other action as `legal` right away.

Let us define

$$q^{\mathrm{BG}} := \lim_{n\to\infty} q(\mathtt{bg}, \mathrm{BG}) = \lim_{n\to\infty} q(\mathtt{b}, \mathrm{BG}) = 41/48;$$

$$q^{\mathrm{B}} := \lim_{n\to\infty} q(\mathtt{b}, \mathrm{B}) = 5/6;$$

and

$$\begin{aligned}\bar{q} &:= \lim_{n\to\infty} \Pr(\phi = \mathrm{BG}|\phi = \mathrm{BG} \text{ or } \mathrm{B}) \times q(\mathtt{b}, \mathrm{BG}) \\ &\quad + \Pr(\phi = \mathrm{B}|\phi = \mathrm{BG} \text{ or } \mathrm{B}) \times q(\mathtt{b}, \mathrm{B}) \\ &= \left(\frac{4v}{3+v}\right) \times \frac{41}{48} + \left(\frac{3-v}{3+v}\right) \times \frac{5}{6}.\end{aligned}$$

Let us define

$$\theta^{\mathrm{BG}} := \lim_{n\to\infty} \Pr(m = 2|\mathrm{BG}) = 7/16;$$

and

$$\theta^{\mathrm{B}} := \lim_{n\to\infty} \Pr(m = 2|\mathrm{B}) = 1/2.$$

Note for future reference that

$$0 < \theta^{\mathrm{BG}}/4 < \theta^{\mathrm{B}}/4 < 1 - q^{\mathrm{BG}} < 1 - \bar{q} < 1 - q^{B} < 1/2. \tag{1}$$

In the subsequent analysis, we consider only generic parameter values. In particular, we ignore the non-generic case where the total litigation cost associated with the `fairness test`, $l$, equals to any of the values of $1 - q^{\mathrm{BG}}$, $1 - \bar{q}$ and $1 - q^{\mathrm{B}}$. Similarly, we ignore the non-generic case where the precision of the `fairness test`, $p$, equals to any of the values of $q^{\mathrm{B}}$, $q^{\mathrm{BG}}$, $1 - \theta^{\mathrm{B}}/4$ and $1 - \theta^{\mathrm{BG}}/4$.

LEMMA 3. *In any equilibrium* $(\sigma_L^*, \sigma_J^*)$, *if* $\sigma_L^*(\mathrm{BG}) = \varnothing$, *then* $\sigma_L^*(\mathrm{B}) = \varnothing$.

Lemmas 1–3 narrow the set of possible equilibria down to three possibilities:

(*i*) $\sigma_L^*(\mathrm{BG}) = \mathtt{bg}$ and $\sigma_L^*(\mathrm{B}) = \mathtt{b}$; we call this a communicative equilibrium. It is the only equilibrium where a legislator with information set BG writes every action, good or bad, he is aware of into the law. However, he writes the good action into the law not so much because he wants to explain the boundary between good and bad behaviour in greater detail – such a benefit would have been easily outweighed by the writing cost. Instead, he is more motivated by the desire to signal his higher awareness;

(*ii*) $\sigma_L^*(\mathrm{BG}) = \sigma_L^*(\mathrm{B}) = \mathtt{b}$; we call this a regular equilibrium. The legislator simply lists all the bad action(s) he is aware of; and

(*iii*) $\sigma_L^*(\mathrm{B}) = \varnothing$ (in which case we must also have $\sigma_J^*(\mathtt{b}) = \mathtt{legal}$, otherwise a legislator with information set B would have deviated to writing the law `b`); we call this a loophole equilibrium. A legislator with information set B refrains from listing the bad action he is aware of, because he is aware that there may be a second bad action that he is not yet aware of, and he does not want the judge to rule that second bad action as `legal` right away. In a sense, the longer law `b` creates a loophole, which protects the second bad action from being prosecuted. The simpler law $\varnothing$ (paradoxically) eliminates that loophole, and the second bad action will be ruled as `illegal` with probability $p > 1/2$.

We shall give the range of parameter values for each of these equilibria to exist. Recall that we consider only generic parameter values. In particular, we ignore the non-generic case where $l = 1 - q^{\mathrm{BG}}$, $1 - \bar{q}$, or $1 - q^{\mathrm{B}}$, or where $1 - p = 1 - q^{\mathrm{B}}$, $1 - q^{\mathrm{BG}}$, $\theta^{\mathrm{B}}/4$, or $\theta^{\mathrm{BG}}/4$.

Proposition 1. *In a communicative equilibrium, we must have* $\sigma_J^*(\texttt{bg}) = \texttt{legal}$ *and* $\sigma_J^*(\texttt{b}) = \texttt{fairness test}$. *A communicative equilibrium exists iff* $1 - q^{\mathrm{BG}} < 1 - p$ *and* $l < 1 - q^{\mathrm{B}}$.

Proposition 2. *A regular equilibrium exists iff either* (1) $1 - p < 1 - q^{\mathrm{BG}}$ *and* $l < 1 - \bar{q}$ (*in which case* $\sigma_J^*(\texttt{b}) = \texttt{fairness test}$); *or* (2) $\theta^{\mathrm{B}}/4 < 1 - p$ *and* $1 - \bar{q} < l$ (*in which case* $\sigma_J^*(\texttt{b}) = \texttt{legal}$).

Proposition 3. *A loophole equilibrium exists iff* $1 - p < \theta^{\mathrm{B}}/4$ *and* $1 - q^{\mathrm{BG}} < l$. *In a loophole equilibrium, we must have* $\sigma_J^*(\texttt{bg}) = \sigma_J^*(\texttt{b}) = \texttt{legal}$. *Furthermore,* $\sigma_L^*(\mathrm{BG}) = \texttt{b}$ *if* $1 - p \in (\theta^{\mathrm{BG}}/4, \theta^{\mathrm{B}}/4)$ (*in which case we call the equilibrium a weak loophole equilibrium*), *and* $\sigma_L^*(\mathrm{BG}) = \texttt{fairness test}$ *if* $1 - p < \theta^{\mathrm{BG}}/4$ (*in which case we call the equilibrium a strong loophole equilibrium*).

The parameter range for each of these three kinds of equilibrium to exist is depicted schematically in Figure 1. From Figure 1, we can see that an equilibrium exists for every parameter combination. The loophole equilibrium is the unique equilibrium in some region and co-exists with the regular equilibrium in another region.

The loophole equilibrium formalises the argument of Iredell. A legislator with information set B (and sometimes a legislator with information set BG as well) refrains from writing the bad action he is aware of into the law, because he understands that a more detailed law would prompt the judge to rule any other action not mentioned in the law as `legal` right away and, hence, in effect, protect any bad action he is not yet aware of, whereas a simpler law would invite the judge to deliberate seriously before ruling on any action, including any bad action he is not yet aware of. Such behaviour on the part of the judge is also well justified. A simpler law, on the one hand, gives the judge little guidance, and leaves the judge little choice but to deliberate seriously. A more detailed law, on the other hand, suggests that the law is likely authored by a legislator with higher awareness, who in turn is less likely to leave out any bad action from the law, hence justifying the judge's ruling any other action not mentioned in the law as `legal` right away.

The loophole equilibrium is not a phenomenon of coordination failure. In some parameter ranges it is the unique equilibrium and, hence, one cannot dismiss it as an implausible selection out of multiple equilibria. Nor is the loophole equilibrium built on pathological out-of-equilibrium beliefs, as our definition of an equilibrium has the intuitive criterion as an integral part.

The loophole equilibrium also highlights a potential difficulty in the argument of Iredell. Suppose $1 - q^{\mathrm{BG}} < l < 1 - q^{B}$ and hence the judge would optimally

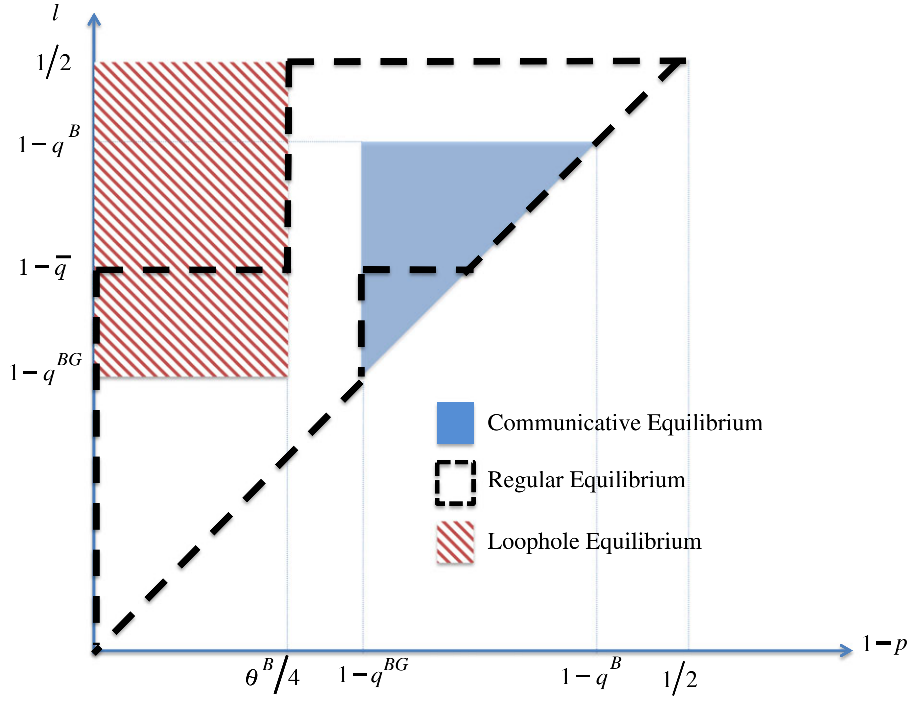


Fig. 1. *The Parameter Range for Each of the Three Kinds of Equilibrium*

choose $\sigma_J(\mathtt{b})$ = `fairness test` if she believes that the law is authored by a legislator with information set B but would optimally choose $\sigma_J(\mathtt{b})$ = `legal` if she believes that it is authored by a legislator with information set BG. Suppose also $\theta^{\mathrm{BG}}/4 < 1 - p < \theta^{\mathrm{B}}/4$ and hence $\sigma_L(\mathrm{BG}) = \mathtt{b}$ and $\sigma_L(\mathrm{B}) = \varnothing$ in the (weak) loophole equilibrium. Therefore, the judge justifiably infers that the author of the law `b` is a legislator with information set BG, and hence optimally plays $\sigma_J(\mathtt{b})$ = `legal`. A legislator with information set B would have loved to play $\sigma_L(\mathrm{B}) = \mathtt{b}$ if he had a means to tell the judge that he has information set B instead of BG, for then the judge would happily switch to playing $\sigma_J(\mathtt{b})$ = `fairness test`, which would result in a higher utility for him. However, he does not have such a means, because the laws he is allowed to write are artificially limited to only `b` and $\varnothing$ in this model. In particular, he is not allowed in this model to write the law `b+`, which is the same as the law `b` except for the extra sentence of 'I am a legislator with information set B instead of BG'. Would the loophole equilibrium survive if we extend the set of laws `LAWS` to include `b+` as well?

We shall answer this question in the next Section. Before we close this Section, let us summarise what we know about the loophole equilibrium in the following theorem.

THEOREM 1. *An equilibrium exists for every combination of* $(p,l)$ *satisfying* $p \in (1/2,1)$ *and* $l \in (1-p,1/2)$. *For* $(1-p, l) \in (0, \theta^{\mathrm{B}}/4) \times (1-q^{\mathrm{BG}}, 1/2)$, *a loophole equilibrium exists, where*

$$\sigma_L^*(\mathrm{BB}) = \texttt{bb},$$
$$\sigma_L^*(\mathrm{GG}) = \sigma_L^*(\mathrm{B}) = \sigma_L^*(\mathrm{G}) = \varnothing,$$
$$\sigma_L^*(\mathrm{BG}) = \begin{cases} \texttt{b} & \text{if } 1-p \in (\theta^{BG}/4, \theta^{\mathrm{B}}/4) \\ \varnothing & \text{if } 1-p < \theta^{BG}/4 \end{cases},$$
$$\sigma_J^*(\texttt{bb}) = \sigma_J^*(\texttt{bg}) = \sigma_J^*(\texttt{b}) = \texttt{legal},$$
$$\textit{and} \qquad \sigma_J^*(\texttt{gg}) = \sigma_J^*(\texttt{g}) = \sigma_J^*(\varnothing) = \texttt{fairness test}.$$

*For* $(1-p, l) \in (0, \theta^{\mathrm{B}}/4) \times (1-\bar{q}, 1/2)$, *a loophole equilibrium is the unique equilibirum.*

## 5. 'This List is Not Meant to be Exhaustive …'

Here comes the million-dollar question: would Iredell have agreed to include the Bill of Rights in the American Constitution if he had thought of adding the sentence 'this list of rights is not meant to be exhaustive, and hence this Bill should not be interpreted as suggesting that any unlisted rights can be impaired by the government'?

In the light of the discussion at the end of the last Section, this question can be formalised as whether the loophole equilibrium would survive if we extend the set of laws, `LAWS`, to include the law `b+`, a law that is the same as the law `b` except for the extra sentence of 'I am a legislator with informatin set B instead of BG'. Let us assume that the law `b+` can be written by a legislator with information set either BB, BG or B, and costs the same to write as the law `b` does.[8] Let us continue to assume that, if nature chooses the single action mentioned in the law `b+`, the judge is required to rule it as `illegal`. However, if nature chooses an action not mentioned in the law `b+`, the judge has the freedom to choose either to rule it as `legal` or `illegal` right away, or to employ the `fairness test` and then rule accordingly.

This describes a new finite dynamic game. We can define an equilibrium for this new game in the same way as in subsection 3.5. Let us abuse terminology and continue to call any equilibrium with $\sigma_L^*(\mathrm{B}) = \varnothing$ a loophole equilibrium in this new game. The next Theorem, which is the key result of this article, says that a loophole equilibrium exists in this new game exactly when a loophole equilibrium exists in the original game and a loophole equilibrium is the unique equilibrium exactly when a loophole equilibrium is the unique equilibrium in the original game.

Why is it so? Although the proof is a bit tedious, the intuition is actually quite simple. If a mere sentence of 'I am a legislator with information set B instead of BG' can successfully convince the judge that the legislator indeed has information set B instead of BG and hence induce her to deliberate seriously before ruling on any action not mentioned in the law, then even a legislator with information set BG would want to add such an extra sentence as well. Since adding such an extra sentence is equally costly (or costless) for a legislator with information sets BG and B, it cannot by itself serve as a credible signalling device for a legislator with information set B.

[8] The assumption that the law `b+`, which contains an extra sentence, costs no more to write than the law `b` does is not important. The proof of Theorem 2 below goes through without any change if we instead assume that, for example, the laws `b+` costs $2c$ instead of $c$ to write.

This settles our million-dollar question: to the extent that Iredell believed that we were in a loophole equilibrium and hence the Bill of Rights should not be included in the American Constitution, he had no reason to change his mind even if he had thought of adding the sentence 'this list of rights is not meant to be exhaustive, and hence this Bill should not be interpreted as suggesting that any unlisted rights can be impaired by the government'. Other founding fathers of America might disagree with Iredell on the values of parameters $1 - p$ and $l$ but it would be wrong to dismiss his concern as illogical.

THEOREM 2. *When a loophole equilibrium exists in the original game, a loophole equilibrium exists in the new game. When a loophole equilibrium is the unique equilibrium in the original game, a loophole equilibrium is the unique equilibrium in the new game.*

## 6. Relation to the Literature on Incomplete Contracts

To us, the more important difference between our article and the previous literature on strategic contract incompleteness is that we have different definitions of contract incompleteness in mind.

Incomplete contracts is an ambiguous term and has at least three different definitions. The first one refers to contracts that are silent on certain contingencies. The second one refers to contracts that are insufficiently state-contingent (such as constant contracts as in Spier, 1992). The third one refers to contracts that include various forms of option/right/discretion/authority that parties can exercise at later dates.[9] These definitions are often at odds with each other. For example, the null contract, which is one of the most insufficiently state-contingent contracts (and hence incomplete in the second sense), is nevertheless complete in the first sense: 'The null contract is complete in that it is absolutely clear what everybody's obligations are: nobody has any!' (Hart and Moore, 1999, p.134) Similarly, a contract that leaves all the control rights to one party may be deemed incomplete in the third sense but not in the first sense.

Our study of Iredell's argument suggests that there may be a fourth definition for incomplete contracts. A contract is more incomplete in this fourth sense if the judge conceives more (subjective) gaps in it, possibly as a result of an unfavourable 'awareness check'. To see how this fourth definition differs from either of the previous three definitions, consider the famous case of ALCOA v. Essex Group, Inc.[10] ALCOA signed a long-term contract with Essex, in which the price Essex was to pay ALCOA for its aluminum would be subject to a price escalator clause based in part on the wholesale price index for industrial commodities (WPI). When later on ALCOA found that the WPI did not rise as fast as its production costs, it reneged. It claimed that the

[9] If we take the mechanism-design perspective and think of contracts as message games, then a contract that leaves all the control rights to one party corresponds to a message game with an outcome function that depends on only one party's messages but not the others'. Less extreme allocations of control rights, such as giving one party the authority over some decisions while giving another party the discretion over some other actions, can be thought of as similar measurability restrictions on the message game's outcome function.

[10] 499 F. Supp. 53 (W.D. Pa 1980).

event that the WPI failed to track production costs was an unforeseen contingency. The judge accepted this argument and released ALCOA from its obligation.

The long-term contract signed by ALCOA probably does not qualify as an incomplete contract in the first sense: what else can be more complete than an explicit function of a publicly available statistics such as the WPI? Similarly, it is neither a null contract nor a constant contract,[11] nor does it grant discretionary power to any party, and hence it probably does not qualify as an incomplete contract in the second or third sense either. Nevertheless, it is incomplete in the fourth sense, because the judge conceived a gap in it – the judge considered the contract as silent on the contingency where the WPI failed to track ALCOA's production costs.

Note that this gap is subjective rather than objective. Objectively, the contract contains no gap, because aluminum price was still well defined as an explicit function of the WPI even in the contingency where the WPI failed to track production costs. In this sense, the fourth definition of incomplete contracts is a modification of the first, with objective gaps replaced by subjective ones. But, this modification makes a big difference: while Hart and Moore (1999) are hard-pressed to find any contract that is incomplete in the first sense, contracts that are incomplete in the fourth sense abound. In fact, even artificial examples of incomplete contracts in legal textbooks are incomplete only in the fourth instead of the first sense. For instance, in Posner (1998), one of the examples of incomplete contracts involves a contract that states that the crew are to work on the ship until it arrives at the final destination. This contract allegedly is incomplete and contains the following gap: it is 'notably' silent on whether the crew have the same obligation if a war breaks out in the destination country. Since the contract is actually no more silent on wars than on any other contingencies, such a gap is more a subjective one conceived by the textbook author. Incidentally, since the textbook author is also a prominent judge,[12] this example also demonstrates that the fourth definition of incomplete contracts is closer to what judges have in mind.

The fourth definition of incomplete contracts also bears more connections with the second definition than the first definition does. For example, constant contracts that are deemed incomplete in the second sense (Spier, 1992) likely will also be seen as being silent on more contingencies by the judge, because the contracting parties will be perceived as having lower awareness. However, this correlation is not perfect. Theoretically, even a constant contract can be written in a very complicated way, for example by enumerating a lot of contingencies while repeating the same transaction terms in each of these contingencies. Such a constant contract would not be deemed as incomplete in the fourth sense, because it signals a high awareness of the contracting parties.

[11] Null contract has an important role in the literature on incomplete contracts. In particular, 'foundations' of incomplete contracts often refer to theories of why contracting parties optimally choose null contracts over other contracts. See Che and Hausch (1999), Segal (1999) and Hart and Moore (1999). An exception is Spier (1992), who defines incomplete contracts as constant contracts: contracts that specify the same obligations for all contingencies.

[12] Posner, the textbook author, is also a judge on the United States Court of Appeals for the Seventh Circuit.

## Appendix A. Omitted Proofs

*Proof of Lemma 1.* *J*'s choice between `legal`, `illegal` and `fairness test` depends on $\alpha(\texttt{law})$. *J* would choose `legal` and `illegal` right away if $1 - \alpha(\texttt{law}) < l$ and $\alpha(\texttt{law}) < l$, respectively; and choose `fairness test` otherwise.

Since $\alpha(\texttt{bb}) = 1$, $\alpha(\texttt{gg}) = (n-2)/(2n-2)$, $\alpha(\texttt{g}) = (n-1)/(2n-1)$ and $\alpha(\varnothing) = 1/2$ in any sequential equilibrium, we have $\sigma_J(\texttt{bb}) = \texttt{legal}$, $\sigma_J(\texttt{gg}) = \sigma_J(\texttt{g}) = \sigma_J(\varnothing) = \texttt{fairness test}$ in any sequential equilibrium when $n$ is sufficiently large. According to our definition of an equilibrium, we have $\sigma_J^*(\texttt{bb}) = \texttt{legal}$ and $\sigma_J^*(\texttt{gg}) = \sigma_J^*(\texttt{g}) = \sigma_J^*(\varnothing) = \texttt{fairness test}$.

Note that $\lim_{n\to\infty} q(\texttt{law},\phi) \geq 1/2 > l$ for any $(\texttt{law},\phi)$. Since $\alpha(\texttt{law})$ is a convex combination of the $q(\texttt{law},\phi)$s, we must have $\alpha(\texttt{law}) > l$ in any sequential equilibrium when $n$ is sufficiently large. Therefore, $\sigma_J(\texttt{law}) \neq \texttt{illegal}$ in any sequential equilibrium when $n$ is sufficiently large. According to our definition of an equilibrium, we have $\sigma_J^*(\texttt{law}) \neq \texttt{illegal}$ for any law `law`.

*Proof of Lemma 2.* Consider a legislator with information set GG first. He can choose to write three kinds of laws: `gg`, `g` and $\varnothing$. By Lemma 1, $\sigma_J(\texttt{gg}) = \sigma_J(\texttt{g}) = \sigma_J(\varnothing) = \texttt{fairness test}$ in any sequential equilibrium when $n$ is sufficiently large. Therefore, by writing laws `gg`, `g`, and $\varnothing$, respectively, his utility will be $-(1-2/2n)(1-p) - 2c$, $-(1-1/2n)(1-p) - c$, and $-(1-p)$, respectively, when $n$ is sufficiently large. When $1/cn$ is also sufficiently small, writing the barebone law $\varnothing$ is his unique best response. According to our definition of an equilibrium, we, therefore, have $\sigma_L^*(\mathrm{GG}) = \varnothing$. The proof of $\sigma_L^*(\mathrm{G}) = \varnothing$ follows the same argument.

Then consider a legislator with information set BB. He can choose to write three kinds of laws: `bb`, `b` and $\varnothing$. By Lemma 1, $\sigma_J(\texttt{bb}) = \texttt{legal}$, $\sigma_J(\texttt{b}) \neq \texttt{illegal}$, and $\sigma_J(\varnothing) = \texttt{fairness test}$ in any sequential equilibrium when $n$ is sufficiently large. Therefore, by writing laws `bb`, `b` and $\varnothing$, respectively, his utility will be $-2c$, $-1/4 - c$ (if $\sigma_J(\texttt{b}) = \texttt{legal}$) or $-3(1-p)/4 - c$ (if $\sigma_J(\texttt{b}) = \texttt{fairness test}$), and $-(1-p)$, respectively, when $n$ is sufficiently large. When $c$ is also sufficiently small, writing the law `bb` is his unique best response. According to our definition of an equilibrium, we therefore have $\sigma_L^*(\mathrm{BB}) = \texttt{bb}$.

Note that the proof of $\sigma_L^*(\mathrm{BB}) = \texttt{bb}$ above makes use of *L*'s knowledge that $\sigma_J(\texttt{b}) \neq \texttt{illegal}$ when $n$ is sufficiently large. But we can prove something stronger: even if $L$ entertains the possibility that $\sigma_J(\texttt{b}) = \texttt{illegal}$, writing the law `bb` is still strictly better than writing the law `b`. The former yields utility $-2c$, while the latter yields utility $-1/2 - c$, which is strictly lower for $c$ sufficiently small. This stronger result is useful later on when we use the intuitive criterion to pin down *J*'s belief $\beta_J(\texttt{b})$, because it says that if $L$ deviates to $\sigma_L(\mathrm{BB}) = \texttt{b}$, such a deviation cannot be rationalised by any belief about $\sigma_J(\texttt{b})$.

*Proof of Lemma 3.* Suppose, contrary to the Lemma, $\sigma_L^*(\mathrm{BG}) = \varnothing$ and $\sigma_L^*(\mathrm{B}) = \texttt{b}$. By Lemma 1, $\sigma_J^*(\varnothing) = \texttt{fairness test}$ and $\sigma_J^*(\texttt{bg}),\sigma_J^*(\texttt{b}) \neq \texttt{illegal}$. Suppose $\sigma_J^*(\texttt{b}) = \texttt{fairness test}$ as well. Then there exists a sequential equilibrium with $\sigma_J(\texttt{b}) = \sigma_J(\varnothing) = \texttt{fairness test}$ and $\sigma_L(\mathrm{BG}) = \varnothing$ for any $n$ sufficiently large and $1/cn$ sufficiently small. However, when $1/cn$ is sufficiently small, $L$ will profitably deviate to $\sigma_L(\mathrm{BG}) = \texttt{b}$, contradicting the existence of such a sequential equilibrium. Therefore, we must have $\sigma_J^*(\texttt{b}) = \texttt{legal}$.

Therefore, there exists a sequential equilibrium with $\sigma_L(\mathrm{BG}) = \varnothing$, $\sigma_L(\mathrm{B}) = \texttt{b}$, $\sigma_J(\texttt{b}) = \texttt{legal}$, and $\sigma_J(\varnothing) = \texttt{fairness test}$ for any $c$ and $1/cn$ sufficiently small. Consider a legislator with information set B. Writing the law `b` yields utility approximately $-\theta^{\mathrm{B}}/4 - c$ when $n$ is sufficiently large, whereas writing the barebone law $\varnothing$ yields utility $-(1-p)$. He will not deviate to $\sigma_L(\mathrm{B}) = \varnothing$ when $n$ is sufficiently large and $c$ is sufficiently small only if $\theta^{\mathrm{B}}/4 < 1 - p$. Since $\theta^{\mathrm{BG}} < \theta^{B}$, we have $\theta^{\mathrm{BG}}/4 < 1 - p$ as well and hence a legislator with information set BG can profitably deviate to $\sigma_L(\mathrm{BG}) = \texttt{b}$ when $n$ is sufficiently large and $c$ is sufficiently small, contradicting the existence of such a sequential equilibrium.

*Proof of Proposition 1.* Suppose a communicative equilibrium exists. Then there exists a sequential equilibrium with $\sigma_L(\text{BG}) = \texttt{bg}$ and $\sigma_L(\text{B}) = \texttt{b}$ for any $c$ and $1/cn$ sufficiently small. In any such sequential equilibrium, $L$ does not deviate to $\sigma_L(\text{BG}) = \texttt{b}$ even with sufficiently small $1/cn$ implies $\sigma_J(\texttt{bg}) \neq \sigma_J(\texttt{b})$. By Lemma 1, $\sigma_J(\texttt{bg}), \sigma_J(\texttt{b}) \neq \texttt{illegal}$ for $n$ sufficiently large. Since $1 - q^{\text{BG}} < 1 - q^{\text{B}}$, $\sigma_J(\texttt{bg}) = \texttt{fairness test}$ would have implied $\sigma_J(\texttt{b}) = \texttt{fairness test}$ as well when $n$ is sufficiently large. Therefore, we must have $\sigma_J(\texttt{bg}) = \texttt{legal}$ and $\sigma_J(\texttt{b}) = \texttt{fairness test}$.

In any such sequential equilibrium, $\alpha(\texttt{bg}) = q(\texttt{bg}, \text{BG}) \rightarrow q^{\text{BG}}$ and $\alpha(\texttt{b}) = q(\texttt{b}, \text{B}) \rightarrow q^{\text{B}}$. Therefore, $\sigma_J(\texttt{bg}) = \texttt{legal}$ and $\sigma_J(\texttt{b}) = \texttt{fairness test}$ are $J$'s best responses to these beliefs iff $1 - q^{\text{BG}} < l < 1 - q^{\text{B}}$.

$\sigma_L(\text{B}) = \texttt{b}$ is obviously the unique best response to $\sigma_J(\texttt{b}) = \sigma_J(\varnothing) = \texttt{fairness test}$ for $c$ sufficiently small. It remains to find the necessary and sufficient condition for $\sigma_L(\text{BG}) = \texttt{bg}$ to be a best response to $\sigma_J$. It suffices to consider only the deviation $\sigma_L(\text{BG}) = \texttt{b}$, as the deviation $\sigma_L(\text{BG}) = \varnothing$ is an inferior deviation for $c$ sufficiently small. For a legislator with information set BG, writing the law $\texttt{bg}$ yields utility approximately $-(1 - q^{\text{BG}}) - 2c$ times the probability that nature chooses an action he is unaware of, whereas writing the law $\texttt{b}$ yields utility $-(1 - p) - c$ times the probability that nature chooses an action not mentioned in the law. When $n$ is sufficiently large, these two probabilities are arbitrarily close to each other. Therefore, $\sigma_L(\text{BG}) = \texttt{bg}$ is a best response for $c$ and $1/cn$ sufficiently small iff $1 - q^{\text{BG}} < 1 - p$.

Since $1 - p < 1 - p + e =: l$, the conditions above simplify into the ones in the proposition.

*Proof of Proposition 2.* A regular equilibrium exists only if there exists a sequential equilibrium with $\sigma_L(\text{BG}) = \sigma_L(\text{B}) = \texttt{b}$ for any $c$ and $1/cn$ small enough. In any such sequential equilibrium, $\alpha(\texttt{bg}) = q(\texttt{bg}, \text{BG}) \rightarrow q^{\text{BG}}$ and $\alpha(\texttt{b}) = \Pr(\phi = \text{BG} | \phi = \text{BG or B}) \times q(\texttt{b}, \text{BG}) + \Pr(\phi = \text{B} | \phi = \text{BG or B}) \times q(\texttt{b}, \text{B}) \rightarrow \bar{q}$. There are three cases to consider: (1) $l < 1 - q^{\text{BG}}$, (2) $l \in (1 - q^{\text{BG}}, 1 - \bar{q})$ and (3) $l > 1 - \bar{q}$.

*Case* (1): $l < 1 - q^{\text{BG}}$. In this case, $\sigma_J(\texttt{bg}) = \sigma_J(\texttt{b}) = \texttt{fairness test}$ in any such sequential equilibrium. $\sigma_L(\text{BG}) = \sigma_L(\text{B}) = \texttt{b}$ is obviously the unique best response to $\sigma_J(\texttt{bg}) = \sigma_J(\texttt{b}) = \sigma_J(\varnothing) = \texttt{fairness test}$ for $c$ sufficiently small.

*Case* (2): $l \in (1 - q^{\text{BG}}, 1 - \bar{q})$. In this case, $\sigma_J(\texttt{bg}) = \texttt{legal}$ and $\sigma_J(\texttt{b}) = \texttt{fairness test}$ in any such equilibrium. $\sigma_L(\text{B}) = \texttt{b}$ is obviously the unique best response to $\sigma_J(\texttt{b}) = \sigma_J(\varnothing) = \texttt{fairness test}$ for $c$ sufficiently small. It remains to find the necessary and sufficient condition for $\sigma_L(\text{BG}) = \texttt{b}$ to be a best response to $\sigma_J$. It suffices to consider only the deviation $\sigma_L(\text{BG}) = \texttt{bg}$, as the deviation $\sigma_L(\text{BG}) = \varnothing$ is an inferior deviation for $c$ sufficiently small. Reversing the argument in the third paragraph in the proof of Proposition 1 establishes that $\sigma_L(\text{BG}) = \texttt{b}$ is a best response for $c$ and $1/cn$ sufficiently small iff $1 - q^{\text{BG}} > 1 - p$.

Since $1 - p < 1 - p + e =: l$, Cases (1) and (2) simplify into Case (*i*) in the proposition.

*Case* (3): $l > 1 - \bar{q}$. In this case, $\sigma_J(\texttt{bg}) = \sigma_J(\texttt{b}) = \texttt{legal}$ in any such sequential equilibrium. It remains to find the necessary and sufficient conditions for $\sigma_L(\text{BG}) = \sigma_L(\text{B}) = \texttt{b}$ to be best responses to $\sigma_J$. It suffices to consider only the deviations $\sigma_L(\text{BG}) = \sigma_L(\text{B}) = \varnothing$, as the deviation $\sigma_L(\text{BG}) = \texttt{bg}$ is an inferior deviation for $1/cn$ sufficiently small. For a legislator with information set BG, writing the $\texttt{b}$ yields utility approximately $-\theta^{\text{BG}}/4 - c$, where $\theta^{\text{BG}}$ is approximately the probability that there is another bad action he is unaware of, and $1/4$ is the probability that such a bad action will be chosen by nature; whereas writing the barebone law $\varnothing$ yields utility $-(1 - p)$. Therefore, $\sigma_L(\text{BG}) = \texttt{b}$ is a best response for $c$ and $1/cn$ sufficiently small iff $\theta^{\text{BG}}/4 < 1 - p$. Similarly, $\sigma_L(\text{B}) = \texttt{b}$ is a best response for $c$ and $1/cn$ sufficiently small iff $\theta^{\text{B}}/4 < 1 - p$. Since $\theta^{\text{BG}} < \theta^{B}$, the conditions above simplify into Case (*ii*) in the Proposition.

*Proof of Proposition 3.* A loophole equilibrium exists only if there exists a sequential equilibrium with $\sigma_L(\text{B}) = \varnothing$ for any $c$ and $1/cn$ small enough. By Lemma 1, $\sigma_J(\texttt{b}) \neq \texttt{illegal}$ and $\sigma_J(\varnothing) = \texttt{fairness test}$ for $n$ sufficiently large. If $\sigma_J(\texttt{b}) = \texttt{fairness test}$ as well, $\sigma_L(\text{B}) = \texttt{b}$

would be a profitable deviation for $c$ sufficiently small, contradicting the existence of such a sequential equilibrium. Therefore, we must have $\sigma_J(\texttt{b}) = \texttt{legal}$ in any such sequential equilibrium, which in turn requires that $1 - \alpha(\texttt{b}) \leq l$ in any such sequential equilibrium. Since $\alpha(\texttt{b}) \in [q(\texttt{b},\text{B}), q(\texttt{b},\text{BG})] \to [q^{\text{B}}, q^{\text{BG}}]$ in any such sequential equilibrium,[13] $\sigma_J(\texttt{b}) = \texttt{legal}$ in any such sequential equilibrium only if $q^{\text{BG}} > 1 - l$. We prove the 'if' part of this sentence later.

In any such sequential equilibrium, $\alpha(\texttt{bg}) = q(\texttt{bg},\text{BG}) \to q^{\text{BG}}$. If $q^{\text{BG}} > 1 - l$, we must also have $\sigma_J(\texttt{bg}) = \texttt{legal}$ in any such sequential equilibrium.

Reversing the argument in Case (3) in the proof of Proposition 2 establishes that $\sigma_L(\text{B}) = \varnothing$ is a best response to $\sigma_J$ for $c$ and $1/cn$ sufficiently small iff $1 - p < \theta^{\text{B}}/4$. If, furthermore, $1 - p < \theta^{\text{BG}}$, then $\sigma_L(\text{BG}) = \varnothing$ as well in any such sequential equilibrium. If, instead, $1 - p \in (\theta^{\text{BG}}/4, \theta^{\text{B}}/4)$, then $\sigma_L(\text{BG}) = \texttt{b}$ in any such sequential equilibrium.

It remains to prove that such $\sigma_J(\texttt{b}) = \texttt{legal}$ is $J$'s best response in any such sequential equilibrium if $1 - q^{\text{BG}} < l$. (The first paragraph in this proof already established the 'only if' part of this sentence.) Consider the belief $\beta_J(\texttt{b})$ that puts probability 1 on $\phi = \text{BG}$. Given such $\beta_J(\texttt{b})$, we have $\alpha(\texttt{b}) = q(\texttt{b},\text{BG}) \to q^{\text{BG}}$ and hence $1 - q^{\text{BG}} < l$ is sufficient for $\sigma_J(\texttt{b}) = \texttt{legal}$ to be $J$'s best response in any such sequential equilibrium. Therefore, it suffices to prove that $\beta_J(\texttt{b})$ is a consistent belief satisfying the intuitive criterion (*à lá* Kreps and Wilson, 1982; Cho and Kreps, 1987). If $\sigma_L(\text{BG}) = \texttt{b}$ (as when $1 - p \in (\theta^{\text{BG}}/4, \theta^{\text{B}}/4)$), then $\beta_J(\texttt{b})$ is an on-equilibrium-path belief and hence *a fortiori* is a consistent belief satisfying the intuitive criterion. If $\sigma_L(\text{BG}) = \varnothing$ (as when $1 - p < \theta^{\text{BG}}/4$), however, then $\beta_J(\texttt{b})$ is an out-of-equilibrium belief and we need to check that the deviation $\sigma_L(\text{BG}) = \texttt{b}$ can indeed be rationalised by some belief of a legislator with information set BG on $\sigma_J(\texttt{b})$. We consider two cases.

Case 1: $l \in (1 - q^{\text{BG}}, 1 - q^{\text{B}})$. In this case, `fairness test` is not a dominated option for a judge facing the law `b` (as it can be optimal if she believes that the legislator of such a law has information set B). Therefore, it is legitimate for a legislator with information set BG to believe that $\sigma_J(\texttt{b}) = \texttt{fairness test}$. Under such a belief, the deviation $\sigma_L(\text{BG}) = \texttt{b}$ is obviously profitable for $c$ sufficiently small.

Case 2: $l > 1 - q^{\text{B}}$. In this case, both `illegal` and `fairness test` are dominated options for a judge facing the law `b`, and hence the only legitimate belief of the legislator is $\sigma_J(\texttt{b}) = \texttt{legal}$. Under such a belief, the deviation to writing the law `b` is not profitable for a legislator with information set BG. However, in this situation, it is also not profitable for a legislator with information set BB or B as well. Hence, the intuitive criterion has no bite on how the judge should form her out-of-equilibrium belief $\beta_J(\texttt{b})$.

*Proof of Theorem 2.* First consider $(1 - p, l) \in (0, \theta^{\text{B}}/4) \times (1 - q^{\text{BG}}, 1/2)$. This is the region where a loophole equilibrium exists in the original game. We explicitly construct an assessment $(\sigma_L, \sigma_J, \beta_J)$ with $\sigma_L(\text{B}) = \varnothing$ and prove that it is a sequential equilibrium for any $c$ and $1/cn$ sufficiently small.

[13] If $\sigma_L(\text{BG}) = \texttt{b}$ (as in the weak loophole equilibrium), then $\alpha(\texttt{b}) = q(\texttt{b}, \text{BG}) \to q^{\text{BG}}$. If $\sigma_L(BG) = \varnothing$ (as in the strong loophole equilibrium), then $\beta_J(\texttt{b})$ is an out-of-equilibrium belief. The intuitive criterion dictates that it assigns zero probability to the information set BB (see the paragraph immediately after the proof of Lemma 2), and hence $\alpha(\texttt{b})$ is a convex combination of $q(\texttt{b},\text{BG}) \to q^{\text{BG}}$ and $q(\texttt{b},\text{B}) \to q^{\text{B}}$.

Consider the following assessment:

$$\sigma_L(\text{BB}) = \texttt{bb},$$
$$\sigma_L(\text{GG}) = \sigma_L(\text{B}) = \sigma_L(\text{G}) = \varnothing,$$
$$\sigma_L(\text{BG}) = \begin{cases} \texttt{b} & \text{if } 1-p \in (\theta^{BG}/4, \theta^{\text{B}}/4) \\ \varnothing & \text{if } 1-p < \theta^{BG}/4 \end{cases},$$
$$\sigma_J(\texttt{bb}) = \sigma_J(\texttt{bg}) = \sigma_J(\texttt{b+}) = \sigma_J(\texttt{b}) = \texttt{legal},$$
$$\sigma_J(\texttt{gg}) = \sigma_J(\texttt{g}) = \sigma_J(\varnothing) = \texttt{fairness test},$$
$$\beta_J(\texttt{bb}) \text{ and } \beta_J(\varnothing) \text{ are obtained from } \sigma_L \text{ by Bayes's Rule,}$$
$$\beta_J(\texttt{bg}), \beta_J(\texttt{b+}), \text{ and } \beta_J(\texttt{b}) \text{ put probability 1 on } \phi = \text{BG},$$
$$\beta_J(\texttt{gg}) \text{ and } \beta_J(\texttt{g}) \text{ put probability 1 on } \phi = \text{GG}.$$

To prove that this is a sequential equilibrium for any $c$ and $1/cn$ sufficiently small, it suffices to check that $\beta_J(\texttt{b+})$ is a consistent belief satisfying the intuitive criterion (*à la* Kreps and Wilson, 1982; Cho and Kreps, 1987), while the rest of the proof is almost the same as the proof of Proposition 3.

Since $\beta_J(\texttt{b+})$ is an out-of-equilibrium belief, we need to check that the deviation $\sigma_L(\text{BG}) = \texttt{b+}$ can indeed be rationalised by some belief on $\sigma_J(\texttt{b+})$. As in the proof of Proposition 3, we consider two cases.

*Case* 1: $l \in (1 - q^{\text{BG}}, 1 - q^{\text{B}})$. In this case, `fairness test` is not a dominated option for a judge facing the law `b+` (as it can be optimal if she believes that the legislator of such a law has information set B). Therefore, it is legitimate for a legislator with information set BG to believe that $\sigma_J(\texttt{b+}) =$ `fairness test`. Under such a belief, the deviation $\sigma_L(\text{BG}) = \texttt{b+}$ is obviously profitable if his equilibrium strategy is $\sigma_L(\text{BG}) = \varnothing$ (as when $1 - p < \theta^{\text{BG}/4}$). It is also profitable if his equilibrium strategy is $\sigma_L(\text{BG}) = \texttt{b}$ (as when $1 - p \in (\theta^{BG}/4, \theta^{\text{B}}/4)$). The deviation yields him utility approximately $-(1 - p)$ times the probability that nature chooses an action not mentioned in the law, whereas his equilibrium strategy yields him utility approximately $-(1 - q^{\text{BG}})$ times the same probability. The deviation is profitable because $1 - p < \theta^{\text{B}}/4 < 1 - q^{\text{BG}}$ according to (1).

Case 2: $l > 1 - q^{\text{B}}$. In this case, both `illegal` and `fairness test` are dominated options for a judge facing the law `b+` and hence the only legitimate belief of the legislator is $\sigma_J(\beta+) = \texttt{legal}$. Under such a belief, the deviation to writing the law `b+` is not profitable for a legislator with information set BG. However, in this situation, it is also not profitable for a legislator with information set BB or B as well. Hence, the intuitive criterion has no bite on how the judge should form her out-of-equilibrium belief $\beta_J(\texttt{b+})$.

This completes the proof of the first half of the theorem. To prove the second half of the theorem, let us consider $(1-p,\, l) \in (0, \theta^{\text{B}}/4) \times (1 - \bar{q},\, 1/2)$. This is the region where a loophole equilibrium is the unique equilibrium in the original game. We prove any equilibrium $(\sigma_L^*, \sigma_J^*)$ in the new game must also have $\sigma_L^*(\text{B}) = \varnothing$.

Consider any sequential equilibrium for $c$ and $1/cn$ sufficiently small. Suppose $\sigma_L(\text{B}) \neq \varnothing$. Then, by exactly the same argument as in the proof of Lemma 3, $\sigma_L(\text{BG}) \neq \varnothing$ as well. Suppose $\sigma_L(\text{BG}) = \sigma_L(\text{BG}) = \texttt{law}$. Then $\alpha(\texttt{law}) \to \bar{q}$. Since $l > 1 - \bar{q}$, $\sigma_J(\texttt{law}) = \texttt{legal}$ in any such sequential equilibrium. However, by reversing the argument in Case (3) of the proof of Proposition 1, $1 - p < \theta^{B}/4$ implies $\sigma_L(\text{B}) = \varnothing$ is a profitable deviation for $n$ large enough, a contradiction.

Therefore, we must have $\sigma_L(\text{BG}) = \texttt{law}^{\text{BG}} \neq \texttt{law}^{\text{B}} = \sigma_L(\text{B})$ for some $\texttt{law}^{\text{BG}}, \texttt{law}^{\text{B}} \in \{\texttt{bg}, \texttt{b+}, \texttt{b}\}$. Then $\alpha(\texttt{law}^{\text{BG}}) = q(\texttt{law}^{\text{BG}}, \text{BG}) \to q^{\text{BG}}$. Since $l > 1 - \bar{q} > 1 - q^{\text{BG}}$, we have $\sigma_J(\texttt{law}^{\text{BG}}) = \texttt{legal}$ for $n$ sufficiently large. If $\sigma_J(\texttt{law}^{\text{B}}) = \texttt{fairness test}$, then by reversing the argument in the third paragraph in the proof of Proposition 1, $\sigma_L(\text{BG}) = \texttt{law}^{\text{B}}$ would have been a profitable deviation for a legislator with information set BG for $c$ and $1/cn$ sufficiently small, because $1 - p < \theta^{B}/4 < 1 - q^{BG}$ according to (1). Therefore, we must have $\sigma_J(\texttt{law}^{B}) = \texttt{legal}$ as well. But, then, by the same argument in the proof of Proposition 3, $1 - p < \theta^{B}/4$ implies that $\sigma_L(B) = \varnothing$ is a profitable deviation for a legislator with information set B for $c$ and $1/cn$ sufficiently small, contradicting the supposition that $\sigma_L(B) \neq \varnothing$. This completes the proof of the second half of the Theorem.

*Chinese University of Hong Kong*
*Northwestern University*

*Submitted: 28 December 2012*
*Accepted: 18 July 2014*

## References

Bernheim, B.D. and Whinston, M.D. (1998). 'Incomplete contracts and strategic ambiguity', *American Economic Review*, vol. 88(4), pp. 902–32.
Bolton, P. and Faure-Grimaud, A. (2010). 'Satisficing contracts', *Review of Economic Studies*, vol. 77(3), pp. 937–71.
Che, Y.K. and Hausch, D.B. (1999). 'Cooperative investments and the value of contracting', *American Economic Review*, vol. 89(1), pp. 125–47.
Cho, I.K. and Kreps, D.M. (1987). 'Signaling games and stable equilibria', *Quarterly Journal of Economics*, vol. 102(2), pp. 179–221.
Ederer, F., Holden, R. and Meyer, M. (2011). 'Gaming and strategic ambiguity in incentive provision', Working Paper, UCLA, University of Chicago, and Oxford University.
Ehrlich, I. and Posner, R.A. (1974). 'An economic analysis of legal rulemaking', *Journal of Legal Studies*, vol. 3 (1), pp. 257–86.
Hart, O.D. and Moore, J. (1999). 'Foundations of incomplete contracts', *Review of Economic Studies*, vol. 66(1), pp. 115–38.
Jehiel, P. and Newman, A. (2012). 'Loopholes and the evolution of contract form', mimeo, Boston University.
Kaplow, L. (1992). 'Rules versus standards: an economic analysis', *Duke Law Journal*, vol. 42(3), pp. 557–629.
Kreps, D.M. and Wilson, R. (1982). 'Sequential equilibria', *Econometrica*, vol. 50(4), pp. 863–94.
Posner, R.A. (1998). *Economic Analysis of Law*, 5th edn, New York: Aspen Law and Business.
Scalia, A. (1998). 'Common-law courts in a civil-law system: the role of united states federal courts in interpreting the constitution and laws', in (A. Gutmann, ed.), *Matter of Interpretation*, pp. 3–47, Princeton, New Jersey: Princeton University Press.
Scott, R.E. and Triantis, G.G. (2006). 'Anticipating litigation in contract design', *Yale Law Journal*, vol. 115(4), pp. 814–79.
Segal, I. (1999). 'Complexity and renegotiation: a foundation for incomplete contracts', *Review of Economic Studies*, vol. 66(1), pp. 57–82.
Spier, K. (1992). 'Incomplete contracts and signalling', *RAND Journal of Economics*, vol. 23(3), pp. 432–43.
Tirole, J. (2009). 'Cognition and incomplete contracts', *American Economic Review*, vol. 99(1), pp. 265–94.